Course Code: 23HS0831 **R23**

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY: PUTTUR

(AUTONOMOUS)

Siddharth Nagar, Narayanavanam Road – 517583

Subject with Code: Differential Equations & Vector Calculus (23HS0831) **Course & Branch**: B.Tech - Common to all

Year & Sem: I-B.Tech & II-Sem **Regulation:** R23

UNIT –I

DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

1	a) Find the Integrating Factor of $\frac{dy}{dx} + y = x$	[L3][CO1]	**[2M]**
	b) Find the Integrating Factor of $\frac{dy}{dx}(x^2y^3 + xy) = 1$	[L3][CO1]	**[2M]**
	c) Verify the exactness of the differential equation $2xydy - (x^2 - y^2 + 1)dx = 0$	[L4][CO1]	**[2M]**
	d) State Newton's law of cooling.	[L1][CO1]	**[2M]**
	e) State Newton's Law of Natural growth and decay.	[L1][CO1]	**[2M]**
2	a) Solve $\boldsymbol{x\frac{dy}{dx} + y = logx}$.	[L3][CO1]	**[5M]**
	b) Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$	[L3][CO1]	**[5M]**
3	a) Solve $(1 + y^2)dx = (tan^{-1}y - x)dy$	[L3][CO1]	**[5M]**
	b) Solve $(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2$	[L3][CO1]	**[5M]**
4	a) Solve $\boldsymbol{x\frac{dy}{dx} + y = x^3y^6}$	[L3][CO1]	**[5M]**
	b) Solve $\frac{dy}{dx} + y.tanx = y^2\, secx$	[L3][CO1]	**[5M]**
5	a) Solve $(2x - y + 1)dx + (2y - x - 1)dy = 0$	[L3][CO1]	**[5M]**
	b) Solve $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$	[L3][CO1]	**[5M]**
6	a) Solve $\frac{dy}{dx} + \frac{ycosx+siny+y}{sinx+xcosy+x} = 0$	[L3][CO1]	**[5M]**
	b) Solve $(x^2\text{-}ay)dx = (ax\text{-}y^2)dy$	[L3][CO1]	**[5M]**
7	a) Solve $x^2ydx\text{-}(x^3 + y^3)dy = 0$	[L3][CO1]	**[5M]**
	b) Solve $y(x^2y^2 + 2)dx + x(2 - 2x^2y^2)dy = 0$	[L3][CO1]	**[5M]**
8	A body is originally at 80^0 C and cools down to 60^0 C in 20 min. If the temperature of the air is 40^0 C, find the temperature of the body after 40 min.?	[L3][CO1]	**[10M]**
9	The temperature of a body drops from 100^0 C to 75^0 C in 10 minutes when the surrounding air is 20^0 C. What will be its temperature after half-an-hour? When will the temperature be 25^0 C ?	[L3][CO1]	**[10M]**
10	The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ *hour* ?	[L1][CO1]	**[10M]**
11	An inductance of 3H and a resistance of 12Ω are connected in series with an e.m.f of 90 V. If the current is zero when t=0, what is the current at the end of 1 sec?	[L1][CO1]	**[10M]**

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UNIT –II

LINEAR DIFFERENTIAL EQUATIONS OF HIGHER ORDER (CONSTANT COEFFICIENTS)

1	a) Solve $\frac{d^2y}{dx^2} - a^2y = 0$	[L3][CO2]	**[2M]**
	b) Find the Particular Integral of $(D^2 + 3D + 2)y = e^{4x}$	[L3][CO2]	**[2M]**
	c) Define Wronskian of functions of $y_1\ and\ y_2$.	[L1][CO2]	**[2M]**
	d) What is the formula of L-C-R Circuit with e.m.f?	[L1][CO2]	**[2M]**
	e) Define Simple Harmonic motion.	[L1][CO2]	**[2M]**
2	a) Solve $(D^2 + 5D + 6)y = e^x$	[L3][CO2]	**[5M]**
	b) Solve $(D^2 - 4D + 3)y = 4e^{3x}$ given ; $y(0) = -1, y^1(0) = 3$.	[L3][CO2]	**[5M]**
3	a) Solve $(D^2 - 3D + 2)y = cos3x$	[L3][CO2]	**[5M]**
	b) Solve $(D^2 - 4D)y = e^x + sin3x.cos2x$	[L3][CO2]	**[5M]**
4	a) Solve $(D^2 + D + 1)y = x^3$	[L3][CO2]	**[5M]**
	b) Solve $(D^2 - 3D + 2)y = xe^{3x} + sin2x$	[L3][CO2]	**[5M]**
5	Solve $\frac{d^2y}{dx^2} + y = e^{-x} + x^3 + e^x sinx$.	[L3][CO2]	**[10M]**
6	a) Solve $(D^2 + 1)y = xsinx$ by the method of variation of parameters.	[L1][CO2]	**[5M]**
	b) Solve $(D^2 + 4)y = tan\, 2\, x$ by the method of variation of parameters.	[L3][CO2]	**[5M]**
7	a) Solve $(D^2 - 2D)y = e^x\, sin\, x$ by the method of variation of parameters.	[L3][CO2]	**[5M]**
	b) Solve $(D^2 + 4)y = Sec2x$ by the method of variation of parameters.	[L3][CO2]	**[5M]**
8	a) Solve $(D^2 + 1)y = Co\, sec\, x$ by the method of variation of parameters.	[L3][CO2]	**[5M]**
	b) Solve $\frac{dx}{dt} = 3x + 2y$: $\frac{dy}{dt} + 5x + 3y = 0$.	[L3][CO2]	**[5M]**
9	a) Solve $\frac{dy}{dx} + y = z + e^x$; $\frac{dz}{dx} + z = y + e^x$.	[L3][CO2]	**[5M]**
	b) Find the current 'i' in the L-C-R circuit assuming zero initial current and charge i , if R=80 ohms, L=20 henrys, C=0.01 farads and E=100 V.	[L3][CO2]	**[5M]**
10	A condenser of capacity 'C' discharged through an inductance 'L' and resistance 'R' in series and the charge 'q' at time 't' satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$. Given that L=0.25 henries, R=250 ohms, C=2x10^{-6} farads, and that when t=0, charge 'q' is 0.002 coulombs and the current $\frac{dq}{dt} = 0$, Obtain the value of 'q' in terms of 't'.	[L3][CO2]	**[10M]**
11	An uncharged condenser of capacity is charged applying an e.m.f $E\ sin\frac{t}{\sqrt{LC}}$ through leads of self-inductance L and negligible resistance. Prove that at time 't', the charge on one of the plates is $\frac{EC}{2}\left[sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}cos\frac{t}{\sqrt{LC}}\right]$.	[L5][CO2]	**[10M]**

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UNIT –III

PARTIAL DIFFERENTIAL EQUATIONS

1	a) Form the Partial differential equation by eliminating the arbitrary constants ‘a’ and ‘b’ form $z = ax + by + a^2 + b^2$.	[L6][CO3]	**[2M]**
	b) Form the Partial differential equation by eliminating the arbitrary constants ‘a’ and ‘b’ from $z = ax + by + \left(\frac{a}{b}\right) - b$.	[L6][CO3]	**[2M]**
	c) Form the Partial Differential Equation by eliminating the arbitrary functions from $z = f(x) + e^y . g(x)$	[L6][CO3]	**[2M]**
	d) Express the Lagrange’s linear form of first order P.D.E.	[L2][CO4]	**[2M]**
	e) Define Homogeneous Linear Partial differential equation with constant coefficients of n^{th} order.	[L1][CO4]	**[2M]**
2	a) Form the Partial Differential Equation by eliminating the constants from $\mathbf{2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}}$	[L6][CO3]	**[5M]**
	b) Form the Partial Differential Equation by eliminating the constants from $(x-a)^2 + (y-b)^2 = z^2 cot^2 \alpha$. where ‘$\alpha$’ is a parameter.	[L6][CO3]	**[5M]**
3	a) Form the Partial Differential Equation by eliminating the constants from $z = a.log\left[\frac{b(y-1)}{(1-x)}\right]$.	[L6][CO3]	**[5M]**
	b) Form the Partial Differential Equation by eliminating the constants from $\boldsymbol{log(az-1) = x + ay + b}$.	[L6][CO3]	**[5M]**
4	a) Form the Partial Differential Equation by eliminating the arbitrary functions from $xyz = f(x^2 + y^2 + z^2)$	[L6][CO3]	**[5M]**
	b) Form the Partial Differential Equation by eliminating the arbitrary functions from $z = xy + f(x^2 + y^2)$	[L6][CO3]	**[5M]**
5	a) Form the P.D.E by eliminating the arbitrary function from $\emptyset\left(\frac{y}{x}, \mathbf{x^2 + y^2 + z^2}\right) = \mathbf{0}$	[L6][CO3]	**[5M]**
	b) Form the P.D.E by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$	[L6][CO3]	**[5M]**
6	a) Solve $\frac{y^2 z}{x} p + xzq = y^2$	[L3][CO4]	**[5M]**
	b) Solve $(z-y)p + (x-z)q = y - x$	[L3][CO4]	**[5M]**
7	Solve $x(y-z)p + y(z-x)q = z(x-y)$	[L3][CO4]	**[10M]**
8	Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$	[L3][CO4]	**[10M]**
9	a) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$	[L3][CO4]	**[5M]**
	b) Solve $r + 6s + 9t = 0$.	[L3][CO4]	**[5M]**
10	Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$	[L3][CO4]	**[10M]**
11	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = cosx\, cos2y$	[L3][CO4]	**[10M]**

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UNIT –IV

VECTOR DIFFERENTIATION

1	a) Define Divergence of a vector.	[L1][CO5]	**[2M]**
	b) Define Solenoidal Vector.	[L1][CO5]	**[2M]**
	c) Find $div\, \vec{r}\ where\ \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$	[L3][CO5]	**[2M]**
	d) Define Irrotational Vector.	[L1][CO5]	**[2M]**
	e) Find $(curl\bar{F})$ given that F = $3xy\bar{i} + 2y^2z\bar{j} + z^2y\bar{k}$ At the point (1-2,-1).	[L3][CO5]	**[2M]**
2	a) Find $grad\ f$ if $f = xz^4 - x^2y$ at a point $(1, -2, 1)$ .Also find $\lvert\nabla f\rvert$	[L3][CO5]	**[5M]**
	b) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then prove that $\nabla r = \frac{\bar{r}}{r}$	[L5][CO5]	**[5M]**
3	a) Find the directional derivative of $2xy + z^2$ at $(1, -1, 3)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$.	[L3][CO5]	**[5M]**
	b) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of normal to the surface $3xy^2 + y = z$ at $(0,1,1)$.	[L3][CO5]	**[5M]**
4	a) Evaluate the angle between the normal to the surface $xy = z^2$ at the points $(4,1,2)$ *and* $(3,3,-3)$.	[L5][CO5]	**[5M]**
	b) Find the maximum or greatest value of the directional derivative of $f = x^2yz^3$ at the point $(2,1,-1)$.	[L3][CO5]	**[5M]**
5	a) Find a unit normal vector to the given surface $z = x^2 + y^2$ at (-1.-2.5).	[L3][CO5]	**[5M]**
	b) Find $div\, curl\ \bar{f}$ for $\bar{f} = yz\bar{i} + zx\bar{j} + xy\bar{k}$	[L3][CO5]	**[5M]**
6	c) Find the divergence of $\bar{f} = (xyz)\vec{i} + (3x^2y)\vec{j} + (xz^2 - y^2z)\vec{k}$.	[L3][CO5]	**[5M]**
	d) Show that $\bar{f} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x - 2z)\vec{k}$ is *solenoidal.*	[L1][CO5]	**[5M]**
7	a) Find $\boldsymbol{div\bar{f}\ \ if\ \bar{f}\ = grad(x^3 + y^3 + z^3 - 3xyz)}$.	[L3][CO5]	**[5M]**
	b) Find the $\boldsymbol{curl}$ of the vector $\boldsymbol{\bar{f} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}}$.	[L3][CO5]	**[5M]**
8	a) Prove that $\bar{f} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$ is *irrotational.*	[L5][CO5]	**[5M]**
	b) Find $curl\ \bar{f}$ if $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[L3][CO5]	**[5M]**
9	a) Find 'a' if $\boldsymbol{\bar{f} = y(ax^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}}$ is *solenoidal.*	[L3][CO5]	**[5M]**
	b) If $\bar{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational then find the constants *a,b and c.*	[L3][CO5]	**[5M]**
10	a) Prove that $div(curl\bar{f}) = 0$.	[L5][CO5]	**[5M]**
	b) Prove that $\nabla(r^n) = n\, r^{n-2}\bar{r}$	[L5][CO5]	**[5M]**
11	a) Prove that $\boldsymbol{curl(\emptyset\bar{f}) = (grad\emptyset) \times \bar{f} + \emptyset(curl\bar{f})}$	[L5][CO5]	**[5M]**
	b) Prove that $\nabla.(\bar{f} \times \bar{g}) = \bar{g}.(\nabla \times \bar{f}) - \bar{f}.(\nabla \times \bar{g})$	[L5][CO5]	**[5M]**

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UNIT –V
VECTOR INTEGRATION

1	a) Define Line integral.	[L1][CO6]	**[2M]**
	b) Define work done by a force.	[L1][CO6]	**[2M]**
	c) State Green's theorem in the plane.	[L1][CO6]	**[2M]**
	d) State Stoke's theorem.	[L1][CO6]	**[2M]**
	e) State Gauss's divergence theorem.	[L1][CO6]	**[2M]**
2	a) If $\bar{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ .Evaluate $\int_c \bar{F}.d\bar{r}$ along the curve $y = x^3$ in xy-plane from $(1,1) to (2,8)$.	[L5][CO6]	**[5M]**
	b) Find the work done by a force $\bar{F} = (2y + 3)\vec{i} + (xz)\vec{j} + (yz - x)\vec{k}$ when it moves a particle from $(0,0,0) to (2,1,1)$ along the curve $x = 2t^2; y = t ; z = t^3$.	[L3][CO6]	**[5M]**
3	If $\bar{F} = (x^2 + y^2)\vec{i} - (2xy)\vec{j}$. Evaluate $\int_c \bar{F}.d\bar{r}$ where 'C' is the rectangle in xy-plane bounded by $y = 0; y = b$ and $x = 0; x = a$.	[L5][CO6]	**[10M]**
4	a) Evaluate $\int_s \bar{F}.\bar{n}ds.$ where $\bar{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and 'S' is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant.	[L5][CO6]	**[5M]**
	b) Evaluate $\int_s \bar{F}.\bar{n}ds.$ where $\bar{F} = 12x^2y\vec{i} - 3yz\vec{j} + 2z\vec{k}$ and 'S' is the portion of the plane $x + y + z = 1$ located in the first octant.	[L5][CO6]	**[5M]**
5	a) If $\bar{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$. Evaluate $\int_v \bar{F}.dv$ where 'V' is the region bounded by the surfaces $x = 0; x = 2 : y = 0; y = 6$ and $z = x^2; z = 4$.	[L5][CO6]	**[5M]**
	b) If $\bar{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ then Evaluate $\int_v \nabla.\bar{F}\, dv$ where 'V' is the closed region bounded by $x = 0; y = 0; z = 0$ and $2x + 2y + z = 4$.	[L5][CO6]	**[5M]**
6	Verify Green's theorem in a plane for $\oint_c (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where 'C' is a square with vertices $(0,0)(2,0)(2,2)$ and $(0,2)$.	[L4][CO6]	**[10M]**
7	a) Apply Green's theorem to evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'C' is the curve enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$.	[L3][CO6]	**[5M]**
	b) Evaluate by Green's theorem $\oint_c (y - sinx)dx + cosx dy$ where 'C' is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $\pi y = 2x$.	[L5][CO6]	**[5M]**
8	Verify Stoke's theorem for the function $\bar{F} = x^2\bar{i} + xy\bar{j}$ integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = a, y = a$.	[L3][CO6]	**[10M]**
9	Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0, y = b$.	[L4][CO6]	**[10M]**
10	Using Gauss's divergence theorem, Evaluate $\iint_s x^3dydz + x^2ydzdx + x^2zdxdy$ where 's' is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0; z = b$.	[L3][CO6]	**[10M]**
11	Verify Gauss's divergence theorem for $\vec{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + z\vec{k}$ taken over the surface of the cube bounded by the planes $x = y = z = a$ and coordinate planes.	[L4][CO6]	**[10M]**

Prepared by: Dept. of Mathematics